One-Dimensional Two-Fluid Model for a Two-Phase Flow

One-Dimensional Two-Fluid Model (I)

- **The one-dimensional, transient, two-phase flow:**
  - a two-phase steam-water mixture
  - noncondensable gases (NCG) in the steam phase
  - a soluble component in the water phase

→ **Two-Fluid Model**

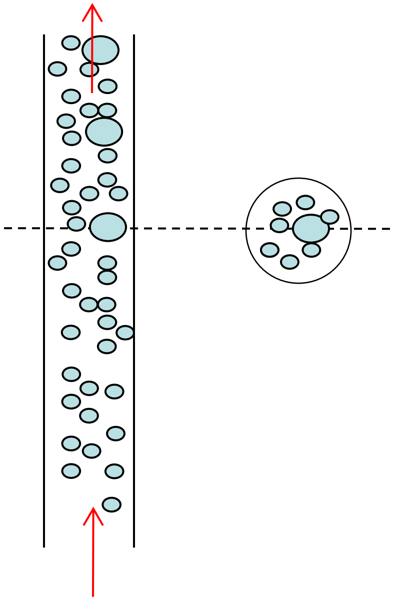
G.B. Wallis (1969)
M. Ishii (1975)
J.M. Delhaye (1981)
R.T. Lahey, Jr. (1977)
M. Ishii and T. Hibiki (2006)

One-Dimensional Two-Fluid Model (II)

❒ Void fraction (Volume fraction of vapor phase)

- Time-averaged volume fraction:

$$\alpha_v = \int_{V_v} dV \Big/ \int_{V} dV = V_v / V$$



- Local volume fraction

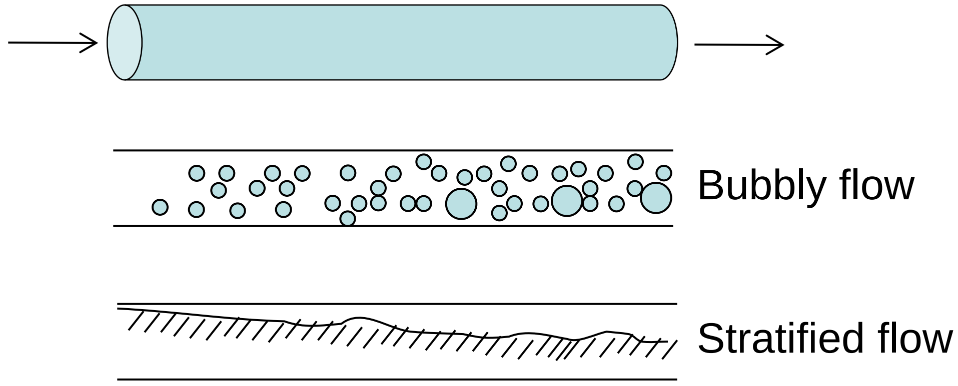
$$\alpha_k = \Delta t_k / \Delta t, \; where \; \Delta t_v + \Delta t_l = \Delta t$$

In the two-fluid formulation, it is assumed that the liquid and vapor phase occupy common space with a volume fraction of α_k under the common pressure gradient.

One-Dimensional Two-Fluid Model (III)

❒ **The two-fluid equations are formulated in terms of volume and time-averaged parameters of the flow:**

- Two-phase mixture is divided into liquid (*l*) and vapor (*v*) phases
- Conservation of mass, energy, and momentum is separately established for each phase.
- The conservation equations for two phases are interconnected by jump conditions at the liquid-vapor interface.

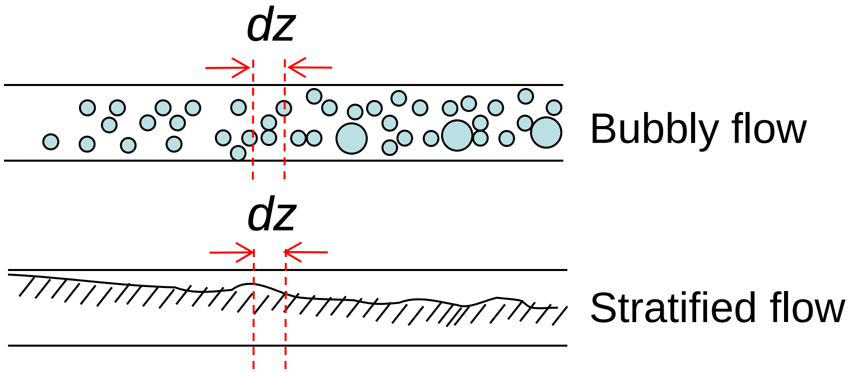


Governing Equations (I)

❖ Mass Conservation

$$\frac{\partial}{\partial t}(\alpha_v \rho_v) + \frac{\partial}{\partial z}(\alpha_v \rho_v v_v) = \Gamma_{iv}$$

$$\frac{\partial}{\partial t}(\alpha_l \rho_l) + \frac{\partial}{\partial z}(\alpha_l \rho_l v_l) = \Gamma_{il}$$



where α_k : Volume fraction of k-phase, $(\alpha_v + \alpha_l = 1)$

ρ_k : Density of k-phase,

v_k : Velocity of k-phase,

Γ_{ik} : Interfacial mass transfer rate of k-phase.

Governing Equations (II)

❖ Momentum Conservation

$$\alpha_v \rho_v \frac{\partial v_v}{\partial t} + \alpha_v \rho_v v_v \frac{\partial v_v}{\partial z} = -\alpha_v \frac{\partial P}{\partial z} + \alpha_v \rho_v B_z - F_{VM} - F_{iv} - F_{Wv} - \Gamma_{iv} v_v$$

$$\alpha_v \rho_l \frac{\partial v_l}{\partial t} + \alpha_l \rho_l v_v \frac{\partial v_l}{\partial z} = -\alpha_l \frac{\partial P}{\partial z} + \alpha_l \rho_l B_z + F_{VM} - F_{il} - F_{Wl} - \Gamma_{il} v_l$$

where P : Pressure,

B_z : Body force (Gravity) of k-phase,

F_{VM} : Virtual mass force,

F_{ik} : Interfacial drag force of k-phase,

F_{Wk} : Wall frictional force of k-phase.

Conservative vs. Non-conservative form

$$\boxed{\frac{\partial}{\partial t}(\alpha_v \rho_v v_v) + \frac{\partial}{\partial z}(\alpha_v \rho_v v_v^2) = -\alpha_v \frac{\partial P}{\partial z} + \alpha_v \rho_v B_z - F_{VM} - F_{iv} - F_{Wv}}$$

Governing Equations (III)

❖ **Energy Conservation**

$$\frac{\partial}{\partial t}(\alpha_v\rho_v U_v)+\frac{\partial}{\partial z}(\alpha_v\rho_v U_v v_v)=-P\,\frac{\partial\alpha_v}{\partial t}-P\frac{\partial}{\partial z}\left(\alpha_v v_v\right)+q_{Wv}+Q_{iv}+DISS_v$$

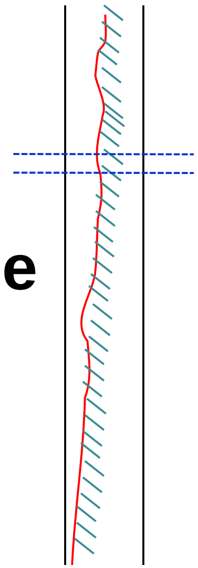
$$\frac{\partial}{\partial t}(\alpha_l\rho_l U_l)+\frac{\partial}{\partial z}(\alpha_l\rho_l U_l v_l)=-P\,\frac{\partial\alpha_l}{\partial t}-P\frac{\partial}{\partial z}\left(\alpha_l v_l\right)+q_{Wl}+Q_{il}+DISS_l$$

where Q_{ik} represents the interface energy transfer due to the phase change and the interfacial heat transfer.

Jump Conditions

❖ **Mass Transfer at the Liquid/Vapor Interface**

$$\Gamma_{iv} = -\Gamma_{il}$$



❖ **Momentum Transfer at the Liquid/Vapor Interface**

$$F_{iv} = -F_{il}$$

❖ **Energy Transfer at the Liquid/Vapor Interface**

$$Q_{iv} = -Q_{il}$$

$$\rightarrow \boxed{[H_{iv}(T^s - T_v) + \Gamma_{iv} h_g^*] = -[H_{il}(T^s - T_l) + \Gamma_{il} h_f^*]}$$

$$\rightarrow \boxed{\Gamma_{iv} = -\frac{H_{iv}(T^s - T_v) + H_{il}(T^s - T_l)}{h_g^* - h_f^*}}$$

Summary of the Governing Equations

$$\frac{\partial}{\partial t}(\alpha_v\rho_v)+\frac{\partial}{\partial z}(\alpha_v\rho_v v_v)=\Gamma_{iv}$$
$$\frac{\partial}{\partial t}(\alpha_l\rho_l)+\frac{\partial}{\partial z}(\alpha_l\rho_l v_l)=\Gamma_{il}$$

$$\Gamma_{iv}=-\Gamma_{il}$$
$$F_{iv}=-F_{il}$$
$$Q_{iv}=-Q_{il}$$

$$\alpha_v\rho_v\frac{\partial v_v}{\partial t}+\alpha_v\rho_v v_v\frac{\partial v_v}{\partial z}=-\alpha_v\frac{\partial P}{\partial z}+\alpha_v\rho_v B_z-F_{VM}-F_{iv}-F_{Wv}-\Gamma_{iv}v_v$$
$$\alpha_v\rho_l\frac{\partial v_l}{\partial t}+\alpha_l\rho_l v_l\frac{\partial v_l}{\partial z}=-\alpha_l\frac{\partial P}{\partial z}+\alpha_l\rho_l B_z+F_{VM}-F_{il}-F_{Wl}-\Gamma_{il}v_l$$

$$\frac{\partial}{\partial t}(\alpha_v\rho_v U_v)+\frac{\partial}{\partial z}(\alpha_v\rho_v U_v v_v)=-P\frac{\partial\alpha_v}{\partial t}-P\frac{\partial}{\partial z}\left(\alpha_v v_v\right)+q_{Wv}+Q_{iv}+DISS_v$$
$$\frac{\partial}{\partial t}(\alpha_l\rho_l U_l)+\frac{\partial}{\partial z}(\alpha_l\rho_l U_l v_l)=-P\frac{\partial\alpha_l}{\partial t}-P\frac{\partial}{\partial z}\left(\alpha_l v_l\right)+q_{Wl}+Q_{il}+DISS_l$$

Mathematical Closure

- **The number of unknowns should be equal to the number of the equations**

  → Equation of State
  → Models and Correlations

Equation of State (EOS)

$$\rho_l = \rho_l(P,U_l)$$

$$T_l = T_l(P,U_l)$$

$$T^s = T^s(P_v)$$

$$\frac{\partial \rho_l}{\partial P} = \frac{C_{p_f} \cdot v_l \cdot \kappa_l - T_l \cdot (v_l \cdot \beta_l)^2}{v_l^2 \left(C_{p_f} - v_l \cdot \beta_l \cdot P \right)}$$

$$\frac{\partial \rho_l}{\partial U_l} = -\frac{v_l \cdot \beta_l}{v_l^2 \left(C_{p_f} - v_l \cdot \beta_l \cdot P \right)}$$

$$\frac{\partial T_l}{\partial P} = \frac{P \cdot v_l \cdot \kappa_l - T_l \cdot v_l \cdot \beta_l}{C_{p_f} - v_l \cdot \beta_l \cdot P}$$

$$\frac{\partial T_l}{\partial U_l} = -\frac{1}{C_{p_f} - v_l \cdot \beta_l \cdot P}$$

$$\frac{\partial T^s}{\partial P} = T_s \frac{v_v^s - v_l^s}{h_v^s - h_l^s}$$

$$\rho_v = \rho_v(P,U_v)$$

$$T_v = T_v(P,U_v)$$

$$\frac{\partial \rho_v}{\partial P} = -\frac{C_{p_v} \cdot v_v \cdot \kappa_v - T_v \cdot (v_v \cdot \beta_v)^2}{v_v^2 \left(C_{p_v} - v_v \cdot \beta_v \cdot P \right)}$$

$$\frac{\partial \rho_v}{\partial U_v} = -\frac{v_v \cdot \beta_v}{v_v^2 \left(C_{p_v} - v_v \cdot \beta_v \cdot P \right)}$$

$$\frac{\partial T_v}{\partial P} = -\frac{P \cdot v_v \cdot \kappa_v - T_v \cdot v_v \cdot \beta_v}{C_{p_v} - v_v \cdot \beta_v \cdot P}$$

$$\frac{\partial T_v}{\partial U_v} = -\frac{1}{C_{p_v} - v_v \cdot \beta_v \cdot P}$$

Under the presence of noncondensable gases, the gas-phase EOSs become different from the above EOSs.

Models & Correlations

❖ **For mathematical closure, models & correlations as well as EOSs are needed:**

- Wall heat transfer
- Wall friction
- Interface heat & mass transfer
- Interface momentum transfer
- Etc.